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SOME PROPERTIES OF FIXED POINT THEOREM IN D-METRIC SPACE

DEEPTI AGARWAL & RUCHIKA TYAGI

Assistant Professors, KITPS Moradabad, Uttar Pradesh, India

ABSTRACT

Some results given we have in D-metric spaces are obtained and the theorems is introduced results are unique and contradictory. The D-metric topology is defined some topological properties of fixed point, Continuous, Completeness and Multivalued of D-metric space.

KEYWORDS: D-Metric Spaces, Theorems, Fixed Point Theorem

INTRODUCTION

Sastri Babu and Naidu [7]Obtained common fixed point theorem for two partially commuting pairs of self maps using a rational inequality. Pagey [2] proved same fixed point theorem using concept of compatible mappings and relative asymptotically regularity. Modi, Tiwari, and Chande [1] tried to find result on D – metric space over a topologycal semi field point theorem for rational inequality Shrivastava and Ghosh [9] has also studied some properties of fixed point theorem in special cases. Rao, Nirmala and Devi [5] Modified some fixed point theorem Vadshoh and gagrini [10] got a common fixed point theorem for pairs of commuting maps on a D- metric space. Rajput Anil and Smriti Arya [3] proved some result on fixed point for θ - contractive mappings in D – metric space.

Sharma and Ravi Dewan [8]extended results of Ganguli and Bondpadhyayto give some fixed point theorem for four mappings in a D – metric space using D – compatibility. Our aim is to consider the inequality used by Rajput & Arya [4] and keeping in view of Shrivastava and Ghosh [9] to obtain some fixed point theorem.

1.1 PRELIMINARIES

Here R+ denotes set of all non –negative real numbers N the set of natural numbers.

$\emptyset R^{+} \rightarrow R^{+}; n \rightarrow \infty Q^{n}(t) = 0, \emptyset(t) < t$

A non –empty set together with X and a function

$D: X \times X \times X \rightarrow R^{+}$called aD- metric on X becomes a D – metric space (X, D), If D satisfies.

1.1.1

(a) $D(x, y, z) \geq 0 \ \forall \ x, y, z \in X$ equality only when $x = y = z$

(b) $D(x, y, z) = \emptyset D(y, x, z) = \cdots$. symmetric

(c) V$D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z) \ \forall \ x, y, z, a \in X$

1.1.2

A sequence $\{x_n\}$ is a D – metric space (X, D) is said to be D –convergent and convergent at a point $x \in X$ if

1.1.3

$$\underset{mnP}{\mathbf{D}} (\mathbf{x_m}, \mathbf{x_n}, \mathbf{n}) = \mathbf{0}$$

1.1.4 (Def)

A complete D – metric space X one in which every D Cauchy sequence converges to a point in X.

1.1.5 (Def)

A subset of S on X is said to be bounded if there exists a constant M > 0 such that D (x, y, z) < M $\forall$ x, y, z $\in$ S then constant M is called the d- bounded on S.

1.1.6 (Def)

A metric X is compact if there exist a finite number of elements $x_1, x_2, \dots\dots.. x_n$ in X such that $X \subset \bigcup_{i=1} B(x, t)$

1.1.7(Def)

A metric space (X,D)is said to be compact D – Metric space if every sequence $\{x_n\}$ in X has convergent subsequence.

1.1.8 (Def)

Let $x_0 \in X$ be fixed and $\in > 0$ is given then we define the open balls $B^*(x_0, \in)$ & $B(x_0, \in)$ in X centered of x_0 of radius $\in$ represented by.

1.1.9

$$B^*(x_0, \in) = \{y \in X: D(x_0, y, z) < \in\} \; \& \; \left\{ \begin{matrix} B(x_0, \in) = \\ \bigcap_{z \in X}(y, z \in X, D(x_0, y, z) < \epsilon) \end{matrix} \right\}$$

Then collection of all open balls $\{ B^*(x, \in): x \in X \; \& \; B(x, \in): x \in X\}$

Defines topologies on X denoted by τ^* and τ respectively.

1.1.10

If B (X) is the collection of all non -empty bounded subsets of a D- metric space (X, D) for A, B, C $\in$ B (X)

1.1.11

Let H (a,b,c) = Z sup {D (a,b,c); $a \in A, b \in B, c \in C$ }then

1.1.12

(a) $H(A,B,C) \geq 0 \; \forall \; A,B,C \in B(X)$ and $H(A,B,C) = 0 \Rightarrow A = B = C$

With a singleton and if A,B,C then $H(A,B,C) =$ perimeter of largest ……. In set A > 0 otherwise H is singleton.

(b) $H(A,B,C) = H(B,C,A) = H(C,A,B) \; \forall \; A,B,C \in$ B (X)

(c) $H(A,B,C) \leq H(A,B,E) + H(A,E,C) + H(E,B,C) \forall \; A,B,C \in$ B (X)

1.1.13 (Def)

Let (X, D) be a D – metric space and CB (X) be the set of all bounded closed subset of X. Let T : X $\to$ $CB(X)$, T is said to be multi valued contraction mapping iff

$H(T_x, T_y, T_z) \leq q \, D(x, y, z) \forall x, y, z \in X \; where \; 0 \leq q < 1$ is a fixed real number.

1.1 14 (Def)

An orbit o(x) multiplication $T: X \to C\ B\ (X)$ as the point x is the sequence $\{X_n\ x_n\ \in Tx_{n-1}\}$ where $x_0 = x$, $0(x)$ is called singular iff

$$D(x_{n+1}, x_{n+2}, x_{n+3}) \le D(x_n, x_{n+1}, x_{n+2})$$

$$D(x_{n+1}, x_{n+2}, x_{n+3}) \le H(Tx_n, Tx_{n+1}, Tx_{n+2})$$

1.1.15 (Def)

A multiplication T is said to be contraction iff for each

$$x_1, x_2, x_3 \in X with x_1 \neq x_2 \neq x_3\ H(Tx_1, Tx_2, Tx_3) \le D(x_1, x_2, x_3)$$

1.1.16 (Def)

A mapping F : (X, D_x) → (y, D_Y) is said to be continuous if u is any open set any then F'u is open in X.

1.2 MAIN RESULT

Let (X,D) be complete bounded D -metric space and $T: X \to C\ B\ (x)$

Be multivalued and orbitally continuous satisfying [4]

1.2.1

$$H(T_x, T_y, T_z) \le \alpha_1 D^*(x, y, z) + \alpha_2 D^*(x, Tx, z) + \alpha_3 D^*(y, Ty, T_z) + \alpha_4 D^* \frac{(y, Ty, T_z)[1 + D^*(x, Tx, z)]}{1 + D^*(x, y, z)}$$

Here

$$D^*(a, b, c) = Inf\{D(a, b, c) : a \in A, b\ \in B, c\ \in C\ \forall A, B, C \in CB(X)\}$$

And $0 < (\alpha_1 + \alpha_2) < 1; 0 < (r - \alpha_3 - \alpha_{4)} < 1; o < r < 1, \alpha_1$ are non negative real numbers then T is a fixed point

Proof

Let x $\in X$ be arbitrary and for sequence $\{x_n\}$, we have $x_{n-1} \in T\ x_n$, n $\in \{0\} \cup N$, for a positive number λ, we find

$H(T_{x_0}, T_{x_1}, T_{x_2}) < \lambda\ \Rightarrow\ D(x_1, x_2, x_0) < \lambda$ And also let

$\lambda = r^{-1}\ H(T_{x_0}, T_{x_1}, T_{x_2})$ Using **(1.3.1)** we get

$$D(x_1, x_2, x_3)\ \le r^{-1}\ H(T_{x_0}, T_{x_1}, T_{x_2})$$

1.2.2

$$D(x_1, x_2, x_3) \le r^{-1} H(T_{x_0}, T_{x_1}, T_{x_2}) \le r^{-1} \left[\alpha_1 D^*(x_0, x_1, x_2) + \alpha_2 D^*(x_0, Tx_0, x_2) + \alpha_3 D^*(x_1, Tx_1, Tx_2) + \alpha_4 D^* \frac{((x_1, Tx_1, Tx_2))[1 + D^*(x_0, Tx_0, x_2)]}{1 + D^*(x_0, x_1, x_2)}\right]$$

Or

$\leq r^{-1}\left[\alpha_1 D(x_0,x_1,x_2)+\alpha_2 D(x_0,x_1,x_2)+\alpha_3 D(x_1,x_2,x_3)+\alpha_4(x_1,x_2,x_3)\frac{[1+D^*(x_0,x_1,x_2)]}{1+D^*(x_0,x_1,x_2)}\right]\leq r^{-1}\left[(\alpha_1+\alpha_2)\right.$ $\left.D(x_0,x_1,x_2)+(\alpha_3+\alpha_4)D(x_1,x_2,x_3)\right]$

Thus finally we obtained

1.2.3

$D(x_1,x_2,x_3)\left[1-r^{-1}(\alpha_3+\alpha_4)\right]\leq(\alpha_1+\alpha_2)r^{-1}D(x_0,x_1,x_2)$

This is reduced to

$D(x_1,x_2,x_3)\leq\left(\frac{\alpha_1+\alpha_2}{r-\alpha_3+\alpha_4}\right)D(x_0,x_1,x_2)$

Take $\left(\frac{\alpha_1+\alpha_2}{r-\alpha_3+\alpha_4}\right)=\delta$

Therefore, we get

1.2.4

$D(x_1,x_2,x_3)\leq\delta D\left(T_{x_1},T_{x_2},T_{x_3}\right)\leq r^{-1}H\left(T_{x_1},T_{x_2},T_{x_3}\right)\leq r^{-1}\left[\alpha_1 D^*(x_1,x_2,x_3)+\alpha_2 D^*(x_1,Tx_1,x_3)+\right.$ $\left.\alpha_3 D^*(x_2,Tx_2,x_3)+D^*(x_2,Tx_2,x_3)\{\frac{1+D^*(x_1,Tx_1,x_3)}{1+D^*(x_1,x_2,x_3)}\})\right]$

Or

$D(x_2,x_3,x_4)\leq r^{-1}\left[\alpha_1 D(x_1,x_2,x_3)+\alpha_2 D(x_1,x_2,x_3)+\alpha_3 D(x_2,x_3,x_4)+\alpha_4 D(x_2,x_3,x_4)\right]$

Finally we find

$D(x_2,x_3,x_4)\left[1-r^{-1}(\alpha_3+\alpha_4)\right]\leq r^{-1}(\alpha_1+\alpha_2)D(x_1,x_2,x_3)$

$D(x_2,x_3,x_4)\leq\left(\frac{\alpha_1+\alpha_2}{1-r^{-1}(\alpha_3+\alpha_4)}\right)D(x_1,x_2,x_3)$

Thus we have

1.2.5

$D(x_2,x_3,x_4)\leq\delta\, D(x_1,x_2,x_3)$ From(1.3.4) and (1.3.5), we get

1.2.6

$D(x_2,x_3,x_4)\leq\delta^2 D(x_1,x_2,x_3)$ Continuing this process we have $D(x_n,x_{n+1},x_{n+2})\leq\delta^n D(x_0,x_1,x_2)$

Since $D(x_0,x_1,x_2)$ is bounded therefore we obtain

1.2.7

$D(x_n,x_{n+1},x_{n+2})\leq\delta^n M$

We can write

$\mathrm{D}\left(x_n,x_{n+p},x_{n+p+q}\right)\leq D\left(x_n,x_{n+p},x_{n+1}\right)+D\left(x_{n+1},x_{n+p},x_{n+p+q}\right)$ zs $+D\left(x_n,x_{n+p},x_{n+1}\right)$

Or equivalently

1.2.8

$D(x_n,x_{n+p},x_{n+p+q}) \leq D(x_n,x_{n+2},x_{n+1}) + D(x_n,x_{n+p},x_{n+2})$ zs$+D(x_{n+2},x_{n+p},x_{n+1}) + D(x_n,x_{n+1},x_{n+p+q})$

$+D(x_{n+1},x_{n+p},x_{n+p+q})$

Which in view of 1.3.7 yields

1.2.9

$D(x_n, x_{n+p}, x_{n+p+q}) \leq 2\,\delta^n M + 2\delta^{n+1} M + 2\delta^{n+2} M +$

$= \leq 2\,\delta^n M.\frac{1}{1-0}, \; \delta^n \to 0$ as n$\to \infty \to 0$

Thus $\{x_n\}$ is D- Cauchy.

Since (X, D) is complete D – metric space. Let $\{x_n\} \to u$. Since T is orbitally continuous $\{Tx_n \to Tu\}$ as $x_n \in Tx_{n-1}\ \forall\ n \in \{0\} \cup N$. Then u $\in Tu$ is a common fixed point of T in X.

From 1.3.1

$$D(u,u,v) \leq r^{-1}\left[\alpha_1 D^*(u, u,v) + \alpha_2 D^*(u, Tu,v) + \alpha_3 D^*(u, Tu,Tv) + \alpha_4 D^*(u, Tu,Tv)\frac{[1+D^*(u,Tu,v)]}{1+D^*(u,u,v)}\right]$$

Which is reduced to

D(u, u, v) $\leq r^{-1}\,[\alpha_1 D(u, u,v) + \alpha_2 D(u, u,v) + \alpha_3 D(u, u,v)\, \alpha_4 D(u, u,v)]$

D(u, u, v) $\leq 0 \Rightarrow u = v$

Hence u is a common fixed point of T in X

1.3 GENERALISED RESULTS

Theorem 1.4.1

If S, T be two self maps then for D – metric space X, X be (S, T) orbitally complete and (S, T) is also orbitally bounded then from Rao and Dev [5] we have S and T have a unique common fixed point.

1.3.1

$$D^*(\delta x, Ty, z) \leq q\begin{bmatrix} max D^*(x,y,z), D^*(x,\delta y,z), D^*(y,Ty,z), \\ D^*(x,Ty,z), D^*(u,\delta y,z) \end{bmatrix}$$

$\forall x, y, \in X$ and $z \epsilon\ [0(S, T,x) \cup 0\ (T, S,y)]$

Proof. S and T have a unique common fixed point also we have

1.3.2

(a) G (x) =min $\begin{bmatrix} D^*(x,\delta x,\delta x), D^*(x,Tx,Tx), D^*(x_1,x_1,\delta x), \\ D^*(x,x,Tx), D^*(x,\delta x,T\delta x), D^*(x,Tx,\delta Tx) \end{bmatrix}$

(b) $H_1(x) = max[D^*(x,\delta x,\delta x) D^*(x,x,\delta x) D^*(x,\delta x,T\delta x)]$

(c) $H_2(x) = max[D^*(x,Tx,Tx) D^*(x,x,Tx)]$

Here we use of the following.

1.3.3

G (u) $\leq lim_{n\to\infty} max\{ H_1(x_{2n}) \to H_2(x_{2n+1})\}$ $x_0 \epsilon X, x_{2n+1} = \delta x_{2n,} x_{2n+2} = Tx_{2n+1,}$

n =0, 2,. & $\{x_n\} \to u$

let us take a sequence as follows.

1.3.4

(a)$x_{2n+1} = \delta x_{2n,}$

$x_{2n+2} = Tx_{2n+1}$ $n=0,1,2$ .....

Since $\{x_n\}$ is D Cauchy sequence. So $\{x_n\}$ converges to u $\epsilon X(G,\ H_1, H_2)$ is a pair orbitally lower semi continuous at u $\epsilon\ X$ then S and T have a unique common fixed point.

1.3.5

G (u) $\leq lim_{n\to\infty} max\{ H_1(x_{2n}), H_2(x_{2n+1})\}$

1.3.6

$$= G(u) \leq lim_{n\to\infty}\left[max\left\{max\left\{\begin{matrix} D^*(x_{2n}, x_{2n+1}, x_{2n+1}), D^*(x_{2n}, x_{2n}, \delta x_{2n}) \\ D^*(x_{2n}, x_{2n+1}, x_{2n+2}) \\ D^*(x_{n+1}, x_{2n+2}, x_{2n+2}) \\ D^*(x_{2n+1}, x_{2n+1}, x_{2n+2}) \\ D^*(x_{n+1}, x_{2n+2}, x_{2n+3}) \end{matrix}\right\}\right\}\right]$$

$since$ $\{x_n\}$ is D Cauchy sequence.

Therefore we obtain

Min **{** $D^*(\delta_2, \delta u, \delta u), D^*(u, Tu, Tu), D^*(u, u, \delta u), D^*(u, u, Tu), D^*(u, u, T\delta u), D^*(u, Tu, \delta Tu)\}$

Thus $\delta u = u$

Or Tu = u

Or $\delta u = u = Tu$

Case I:

Let $\delta u = u$ & $Or\ Tu \neq u$

$D^*(u,\ Tu, u) > 0$

$D^*(u,\ Tu, u) = D^*(\delta u,\ Tu, u)$

$$\leq q\begin{bmatrix} maxD^*(u, u, u), D^*(u, u, u), D^*(u, Tu, u,), D^*(u, Tu, u,), \\ D^*(u, u, u), \end{bmatrix}$$

$\leq q\ max\ D^*(u, Tu, u,)$

$\leq D^*(u, Tu, u,), 0 < q < 1$

It is a contradictory

Case II

If T u = u then one can get Su = u

Thus u is a common fixed point of S & T uniqueness can easily be shown.

$$D^*(u, Tu, v,) = D^*(Su, Tu, v,)$$

$$\leq q\ max \begin{bmatrix} D^*(u, Su, Sv), D^*(u, Tu, Tv), D^*(u, Tu, Sv,), D^*(u, u, Tv,), \\ \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{TSv}\ ), \mathrm{D}^*(\mathrm{u}, \mathrm{Tu}, \mathrm{STv}\ ) \end{bmatrix}$$

$$\leq \mathrm{qmax} \begin{bmatrix} \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v}), \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v}), \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v},), \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v},), \\ \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v}), \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v}) \end{bmatrix}$$

$$\leq \mathbf{q\ max}\ \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v})$$

$$\mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v}) \leq \mathrm{D}^*(\mathrm{u}, \mathrm{u}, \mathrm{v})$$

It is contradictory.

Because u = v

Hence proved.

Result- We have to prove that some properties of fixed point theorem in D-metric space.

My work is total unique and easy to shown the result is properly.

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